OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG

Crystallographic Texture – Approximation and Evolution in Terms of Harmonic Tensors

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Introduction. In sheet metal forming, the final shape of the product does not only depend on the geometry of the forming tools but also on the elastic and plastic anisotropy of the material. In practice, phenomenological models are used to describe these effects. In contrast to models based on crystal plasticity (e.g., Sachs and Taylor models), they involve a small number of internal variables and thus computing times are comparatively short. However, they lack a sound foundation in terms of physics, are often limited to particular cases of anisotropy and do not allow for an evolution beyond these. To overcome these drawbacks, we have developed an alternative homogenization scheme for crystal plasticity, by means of which computing times are reduced by several orders of magnitude as compared to Sachs and Taylor models while providing the same results. This allows for a more accurate but equally efficient constitutive model to be used in future forming simulations.

Problem Definition

The general problem is to reduce the computing time for constitutive models based on crystal plasticity. Our focus is on the evolution and the effects of the crystallographic texture, which is described by means of the orientation distribution function (ODF).

Objectives

We introduce a class of non-negative approximations of the ODF based on a finite number of harmonic tensors. We derive their evolution equations using the concept of a motion and find efficient solution strategies for particular cases. In addition, we investigate the convergence of these solutions.

Cooperation

Prof. Thomas Böhlke (Chair for Continuum Mechanics, Karlsruhe Institute of Technology)

Texture, Fourier Expansion, Motion

Orientation distribution function (ODF)
- volume fraction with similar lattice orientation: f (Q,t) dμ (Q) := dV (Q,t) / V
- normalization: ∫ f (Q,t) dμ (Q) = 1
- non-negativity: f (Q,t) ≥ 0

Fourier expansion on SO_3
- infinite list of base functions B (Q)
- infinite list of Fourier coefficients F (t) := ∫ f (Q,t) ⊗ B (Q) dμ (Q)
- Fourier series expansion f (Q,t) = F (t) · B (Q)

Motion on SO_3
- rigid perfectly plastic single crystals: internal state → lattice orientation Q ∈ SO_3
- homogeneous process prescribing spin + stress / strain-rate (Sachs / Taylor model)
- motion χ - sequence of bijective mappings between initial and current orientations

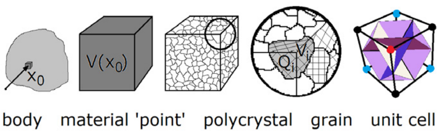

Figure 1. Polycrystal at different length scales.

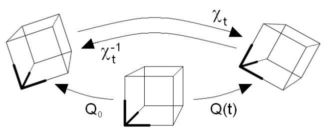

Figure 2. Motion of orientations.

Non-Negative Approximations

- rigid perfectly plastic single crystals + motion on SO_3 ⇒ system of linear ODEs for F
- truncation of infinite Fourier expansion violates non-negativity of the ODF (Fig. 3)
- introduce non-negative function g where f (Q,t) = g (h (Q,t)) and h (Q,t) := H (t) · B (Q)
- motion ⇒ system of quasi-linear ODEs for H
- linear / quadratic approximation (LA/QA) renders linear ODEs: g (h) = h and g (h) = h²

How to Accelerate Approximations

- texture update → matrix exponential applied to vector
- data base for matrix computation, one-time-only costs

Convergence of Stationary Solutions

- convergence = decreasing truncation error for increasing truncation order
- truncation error based on stationary solutions (zero-spin process)
- exact stationary solution: sum of Dirac distributions centered at stationary points
- maximize error w. r. t. process type

Numerical Results

- LA is negative in substantial parts of the orientation space (Fig. 3)
- QAs are non-negative (Fig. 3 – 5)
- QAs converge to the exact solution (Fig. 4)
- critical truncation orders of QAs (Fig. 5) are related to material symmetry as is proven by comparison with different sets of slip systems
- note: numerical results (Fig. 3 – 5) refer to 2D model and Hutchinson's flow rule (p = 15)

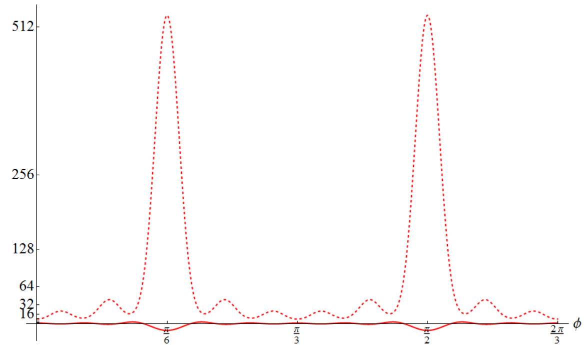

Figure 3. Stationary ODFs for LA (continuous) and QA (dashed) for equal truncation order M = 2 (p+1).

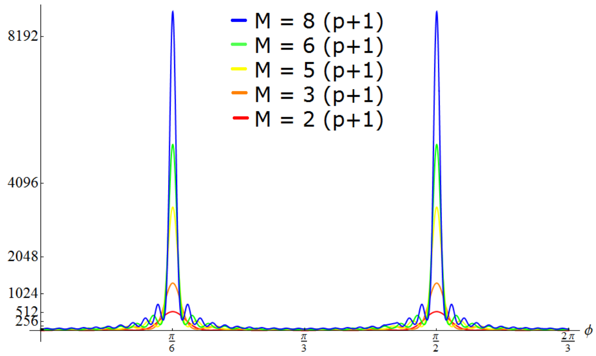

Figure 4. Stationary ODFs according to QAs for different truncation orders

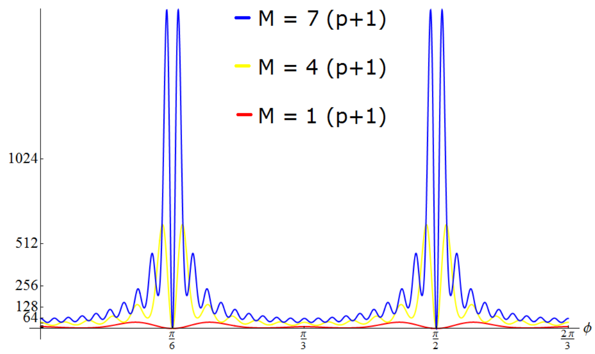

Figure 5. Stationary ODFs according to QAs for critical truncation orders

Results and Discussion

- reduction of computing time by several orders of magnitude as compared to Sachs and Taylor models
- generic scheme for non-negative approximations of the ODF and the respective polycrystal models
- definition of convergence in terms of stationary solutions.
- QA is non-negative, converges to exact solutions, requires approximately the same computing time as LA
- critical truncation orders are due to material symmetries and can be avoided
- no restriction w. r. t. single crystal or texture symmetry

Conclusions

Using the motion and Fourier expansion on SO_3, the quadratic approximation of the ODF gives reasonable predictions for the saturation texture, while the computing time is reduced by several orders of magnitude as compared to the reference models of crystal plasticity by Sachs and Taylor.

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