Finite elements in ferrofluids

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A ferrofluid is a liquid which becomes strongly magnetized in the presence of a magnetic field. Ferrofluids are composed of nanoscale ferroparticles dispersed throughout a carrier liquid. The maximum volume concentration of particles reaches 20%. Despite the ferrofluid is a very stable colloid the particles concentration can be changed in a wide range in presence of a strong magnetic field.
The ferromagnetic seals are available since early 1970s. The combination of magnetic and liquid properties of the material provide very low leakage. Therefore nowadays magnetic fluid seals are mostly used as rotary seals in vacuum applications and other high integrity gas sealing.

Objectives

We want to show that using the complex mathematical model the rotation motion of the shaft keeps magnetic particles dispersed even in presence of a strong magnetic field. For that we have to derive a complex mathematical model, discretize it in a proper way and find a numerical solution. The question of existence and uniqueness of a solution of both the continuous and discrete problems are to be studied. The error estimates are to be obtained.

Cooperation

- **Kristin Held, Robin Gröpler** (Institute of Analysis and Numerics)
  Discussion of numerical methods, their implementation and analysis
- **Anna Girchenko, Mykola Ievdokymov, Oleksandr Prygorniev, Oksana Ozhoga-Maslovskaja** (Institute of Mechanics)
  Discussion of functionality of FEM packages and tools for mesh construction

Problem Statement

On Fig. 1 the schematic view of a ferrofluid seal is presented. It consists of the following parts: 1 – permanent magnet, 2 – core, 3 – magnetic flux concentrator, 4 – shaft, 5 – magnetic fluid, A – region of high pressure, B – region of low pressure.

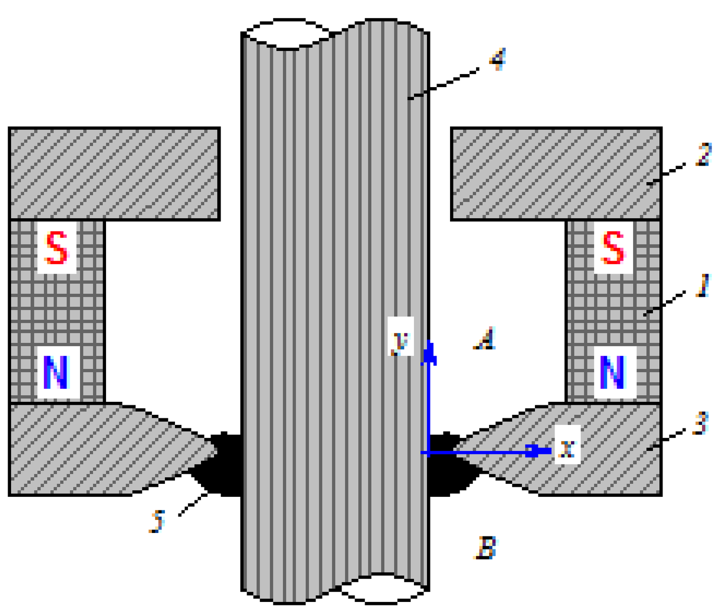

Figure 1. Schematic view of a ferrofluid seal

Main points of problem:
➔Hyperboloidal polar head of the concentrator
➔Gap size is two orders less than the shaft radius.

Ferrofluid in the gap is under the action of the following forces:
- Magnetic force attracts the magnetic fluid
- Centrifugal force of the shaft rotation pushes fluid off the magnet
- Pressure jump between the isolated regions pushes the plug in vertical direction.
- Force balance on the free surface determines its shape

Mathematical Model

Model is basically formulated in 3D. Thanks to axisymmetry of the system and smallness of the gap it is simplified to a 2D planar system in a cross-section. The final model look as follows:

Young-Laplace equation

$$h(X_i, Y_i) + Fr_m \int_0^{X_i} \omega^2 \, dx = C_i - Pm, \quad i = 1,2$$

Decoupled 3-dimensional incompressible Navier-Stokes equation

$$\Delta \omega = 0$$

$$-\frac{1}{\mathrm{Re}} \Delta \upsilon + (\upsilon \cdot \nabla)\upsilon + \nabla p = \delta e_1 \omega^2, \qquad \nabla \cdot \upsilon = 0$$

Convection-diffusion equation

$$-\frac{1}{\mathrm{Pe}} \nabla \left(\nabla C - (\mathrm{Pe}\, \upsilon + \nabla \ln \phi) C \right) = 0, \qquad \frac{1}{|\Omega|} \int_\Omega C = 0$$

+ suitable boundary conditions.

Discretization

We solve the system in the following order:
- Coupled Young-Laplace equation and equation for azimuthal velocity are solved iteratively. The Boundary Element Method is used to discretize the problem for azimuthal velocity.
- Planar Navier-Stokes equation is discretized by $P_2^+ / P_1^{\mathrm{disc}}$ Finite Elements.
- Convection-Diffusion problem is discretized by mixed Finite Element Finite Volume technique.

Numerical method for Convection-Diffusion equation

$$(\nabla u_h, \nabla v_h) + b(u_h, v_h) = 0$$

The first term is discretized by the standard Galerkin method with P_1 finite elements. The second term is obtained by the first order upwinding with special weighting function on a dual domain.

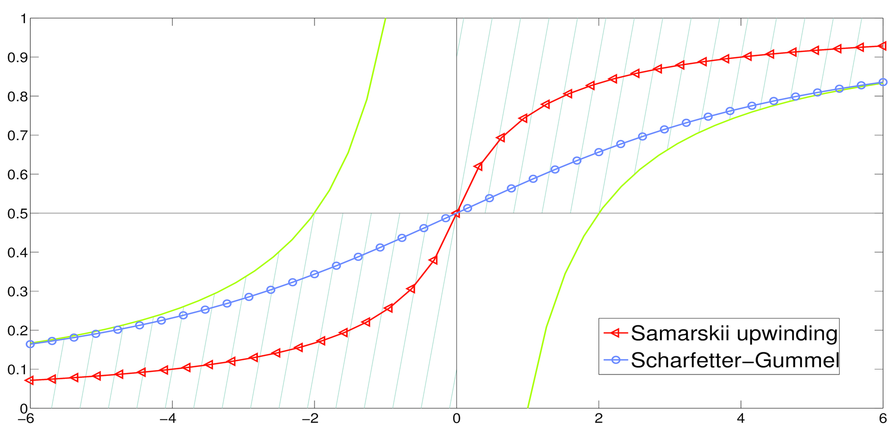

Figure 3. Possible range for choosing the weighting function.

Applied on conforming Delaunay triangulation the method has the following properties:
- There exists a unique solution which is positive
- For sufficiently small mesh sizes the stability is proven

Numerical results

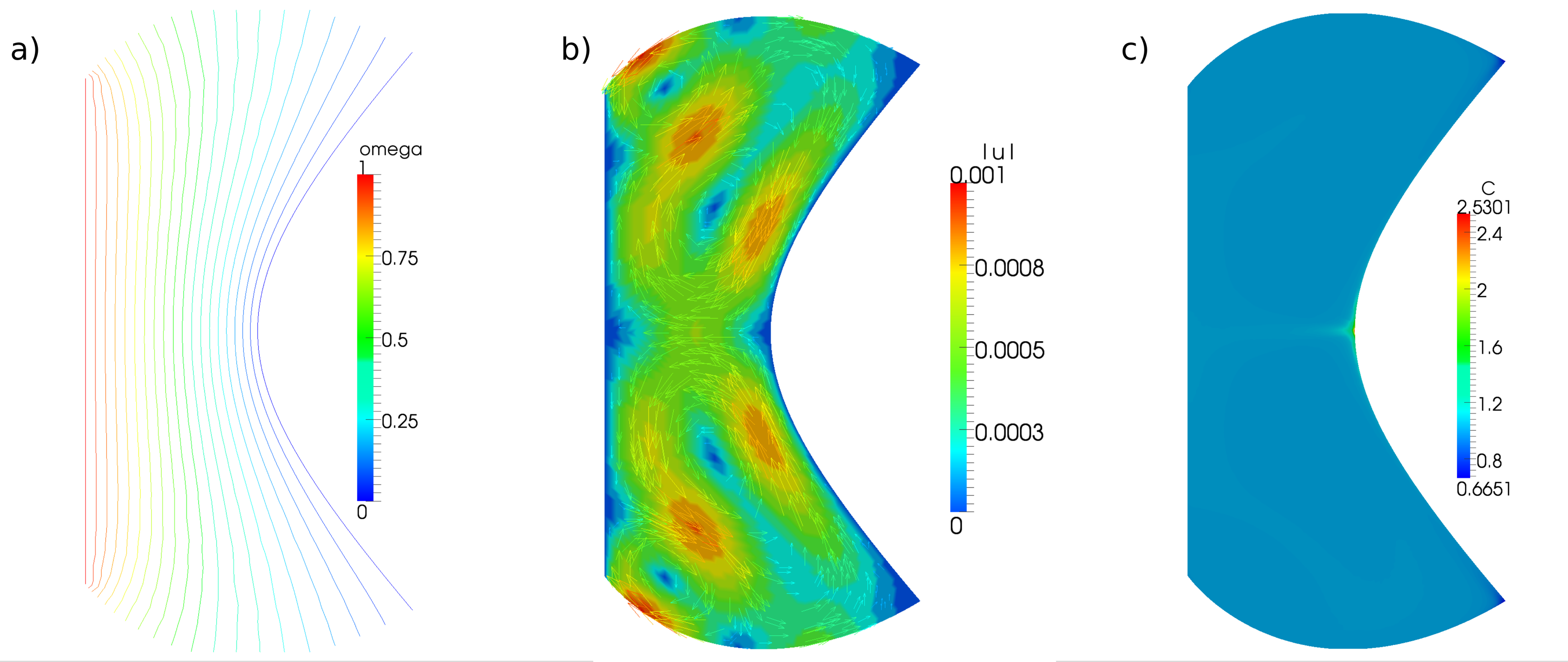

Figure 2. Solutions in computation order: a) azimuthal velocity; b) secondary flow; c) particle concentration.

Summary

- Mathematical model was derived and solution methods were successfully implemented
- Method for Convection-Diffusion equation was obtained and usefull properties were proved.
- The desired effects can be observed

Numerical tests have been performed using the in-house code MooNMD. Specially for that problem were implemented
- Slip boundary conditions for Navier-Stokes equation
- Solution algorithm of Young-Laplace equation
- Discretization of the convection-diffusion equation

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