Material Plasticity

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Introduction.
The aim of this work is to develop a phenomenological finite plasticity theory which describes the evolution of anisotropy. The theory is called **Material Plasticity** (Forest/Parisot (2000), Bertram (2012)) and will be applicable to e.g. fibre reinforced materials. The main difference to a common plasticity theory is the introduction of a second plastic transformation. Therefore the framework of a general finite plasticity theory with two plastic transformations is developed. Some special cases are examined and the case of a Material Plasticity theory is considered more closely. For a later verification of the results of this theory a Representative Volume Element (RVE) for fibre reinforced materials is created with Abaqus and the scripting language Python. As last step the results are compared with experimental data from the Institut für Leichtbau und Kunststofftechnik (ILK) at the TU Dresden.

Problem Definition

Finite elasticity: The stresses depend only on the current deformation. This theory is well understood in the framework of finite strains.

Finite plasticity: The stresses are path-dependend. Until today there are conceptional problems with the definition of plastic deformation.

"There is no general agreement on how it is to be identified, either conceptually or experimentally, in the presence of finite deformations." (Casey/Naghdi 1992)

Objectives

The aim is to develop the theory of Material Plasticity.

Crystal Plasticity:
Under plastic deformations the crystal structure is preserved. The anisotropic axes are not directly fixed to the material

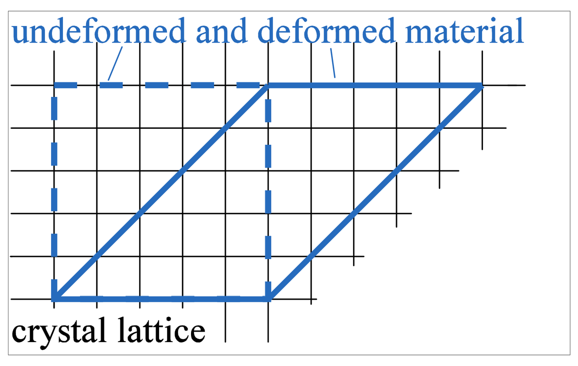

Figure 1: crystal lattice

Material Plasticity:
Under plastic deformations the microstructure deforms together with the material. The anisotropy axes are directly fixed to the material.

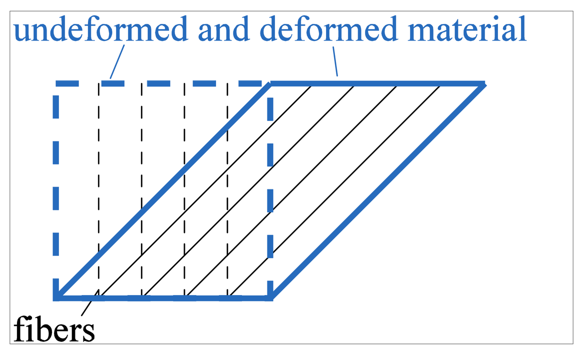

Figure 2: fibre reinforcement

Cooperation

The fibre reinforced composite will be manufactured and tested by the **Institut für Leichtbau und Kunststofftechnik (ILK)** under the direction of **Prof. Hufenbach** at the **TU Dresden**. The experimental data of fibre reinforced material they provide will be compared with our theory of Material Plasticity and the RVE calculations.

TECHNISCHE UNIVERSITÄT DRESDEN

Institut für Leichtbau und Kunststofftechnik

General Plasticity $S = \frac{1}{2}(P_K * \mathbb{K}_0) : \left(C - P_C^{-T} * C_{u0}\right)$

- P_K transforms the elasticities
- P_C transforms the stress-free configuration

$P_K = P_C = 1$	$P_K = P_C \in Inv^+$	$P_K = 1$, $P_C \in Inv^+$	$P_K \in Inv^+$, $P_C = 1$	$P_K, P_C \in Inv^+$
Elasticity	Isomorphic Plasticity	Material Plasticity	Evolving Material	Decoupled Plasticity
Saint Venant-Kirchhoff	crystal structure is preserved	elasticities are fixed to the material	evolving material properties	different evolution of material properties & stress-free placement
elastic materials	materials with crystal structure	fibre reinforced materials	kind of recrystallisation	twinning

What means Material Plasticity?

- with $P_K = 1$ and $P_C \in Inv^+$ the stiffness tetrad $\mathbb{K}_0$ remains the same for arbitrary deformations

Numerical Setup

For validating the results of the new phenomenological Material Plasticity model we use RVE calculations. The RVE for a bidirectional fibre reinforced material is generated using the finite element program Abaqus and its scripting interface Python.

Material	Homogenization	Calculation
fibre radius fibre distance layer distance fibre angle	boundary conditions (linear or periodic) number of unit cells	mesh size elementshape elementtype CPU number

Table1: Categorized input information for the RVE generation and calculation

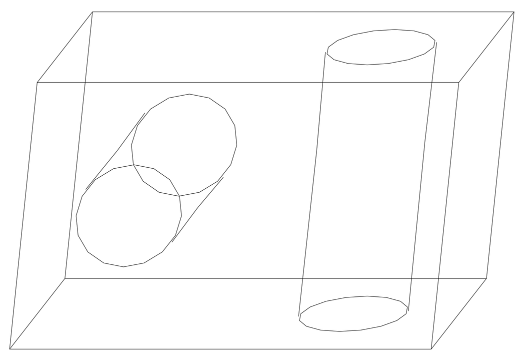

Figure 3: reinforced material

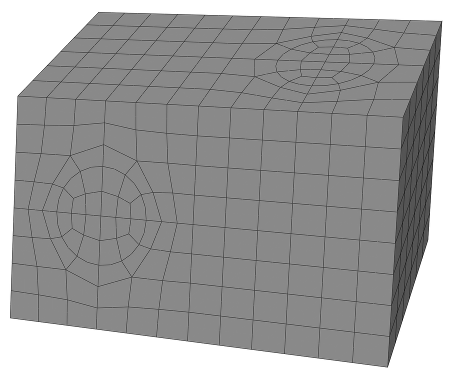

Figure 4: meshed RVE

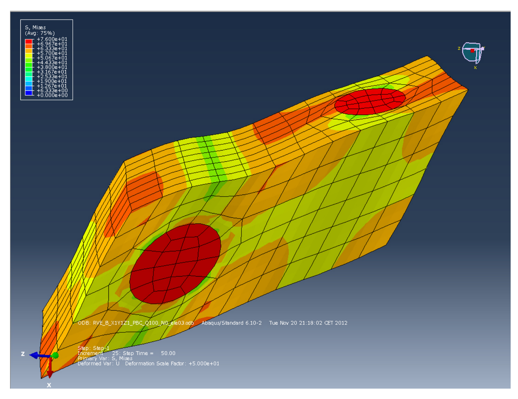

Figure 5: calculation example

- bidirectional reinforced elastic-plastic J2-material
- Cu-fibres $\sigma_F = 75\text{MPa}$, $\text{E} = 123\,000\text{MPa}$, $\nu = 0.35$
- PP-matrix $\sigma_F = 70\text{MPa}$, $\text{E} = 3\,300\text{MPa}$, $\nu = 0.35$
- prescribing deformation gradient **H** or stress tensor **T**

Results and Discussion

Validation
- determine the stiffness tetrad $\mathbb{K}_0$ of the virgin material
- performe a large plastic deformation and unload the material
- determine the stiffness tetrad $\mathbb{K}_1$ of the plastically deformed material
- compare $\mathbb{K}_0$ and $\mathbb{K}_1$ - the following equation should hold:

$$\mathbb{K}_0 = F * \mathbb{K}_1$$

- validation already works for a homogeneous material, future work: reinforced materials

Anisotropic flow rule
- evolution equation for the plastic transformation P_C needed
- anisotropic flow rule necessary
- determining the anisotropic tetrad $\mathbb{A}$ using the yield surface

Yield surface
- calculate the flow stress with the RVE for different loading directions and stress states
- parametrize the loads in the deviatoric stress space using the following decomposition

$$T^{'} = \|T^{'}\| Q\ N(\delta)\, Q^T$$

- a set of almost isotropically distributed orthogonal Qs
- $N(\delta)$ being a tensor function which determines the typ of stress state
- yield indicator: ratio of plastic dissipation over external stress power, see Kraska (1998)

Example calculation setup
- uniaxial tension, set of 600 Qs
- for presenting the yield surface in the 3D space we calculate vectors

$$v_i = \sigma_{Fi} Q_i e_3$$

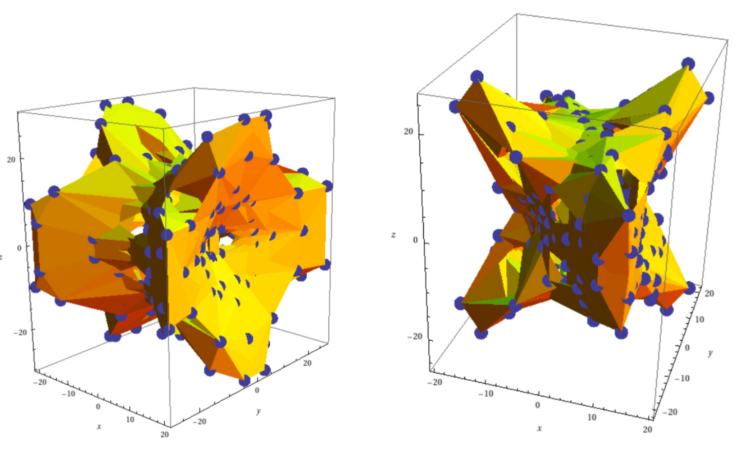

Figure 6: yield surface for uni- and bidirectional reinforcement

Conclusions

Summary
- notation of a General Plasticity theory

$$S = \frac{1}{2}(P_K * \mathbb{K}_0) : \left(C - P_C^{-T} * C_{u0}\right)$$

- introduction of 5 special cases, e.g. Elasticity, Crystal Plasticity and Material Plasticity
- parameterizable RVE generation using Abaqus
- RVE simulations to determine the stiffness tetrad
- anisotropic flow rule
- RVE simulations to determine the yield surface

Future Aims
- validate the stiffness tetrads for reinforced materials
- prove and complete the theory of Material Plasticity
- Implement the theory in Abaqus using the subroutine UMAT
- compare the results of the calculation with the phenomenological model of Material Plastictiy and RVE calculations

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